

III E4

Modul Electricity II

Hall effect

In this experiment, the Hall effect in silver and the anomalous Hall effect in tungsten are investigated by measuring the Hall voltage as a function of the magnetic field for different currents, and the Hall constant is determined in each case. For silver, a negative value of the Hall constant is obtained, indicating charge transport by electrons. The Hall constant for tungsten is found to be positive. This suggests that conduction is mainly due to defect electrons.

Experiment IIE4 - Hall effect

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1.1 Questions for preparation

Good preparation is essential for the success of the experiment. The following questions should be answerable before performing the experiment.

1.1.1 Electricity and magnetism

- What is an electric field?
- How does the Lorentz force work?
- What is meant by induction?
- What happens when direct current flows through a coil? What happens when the direct current is replaced by an alternating current?
- What is the charge carrier density? How does the current I depend on the charge carrier density n ?
- What are typical charge carrier densities for metals, (intrinsic) semiconductors, and insulators?

1.1.2 Hall effect

- What is meant by the Hall effect?
- What is a Hall voltage? What difficulties arise when trying to measure it?
- What can be expected for the Hall voltage when a constant transverse current I and a non-stationary magnetic induction (e.g. $B(t) = B_0 \cdot \sin(\omega t)$) are present?
- How is the Hall coefficient A_H defined?
- The following Hall coefficients A_H can be found in literature:

Material	W	Ag	Cu	Al
Hall constant A_H in [m^3/C]	$7.5 \cdot 10^{-11}$	$-9.0 \cdot 10^{-11}$	$-5.3 \cdot 10^{-11}$	$9.9 \cdot 10^{-11}$

What stands out when looking at the above Hall coefficients A_H ? How can these observations be explained?

- What is the purpose of a Hall probe? How does it work?

1.1.3 Measuring instruments

- What are the purposes of a voltmeter, transformer, power supply, multimeter, and magnetometer?
- What is the approximate internal resistance of a voltmeter? Why is it designed this way?
- What is a reasonable resistance value for an ammeter?
- Why must no current greater than $I = 10\text{ A}$ be applied to a digital multimeter?

1.2 Theory

1.2.1 Derivation of the Hall Voltage

The Hall effect is named after the American physicist Edwin Herbert Hall (1855–1938), who discovered it in 1879.

If we consider a current-carrying conductor in a magnetic field $\vec{B}$, the charge carriers moving within it are subject to the Lorentz force

$$\vec{F}_L = Q \cdot \vec{v} \times \vec{B} \quad (1.1)$$

This force causes the charge carriers to be deflected perpendicular to both the direction of motion and the magnetic field, pushing them toward the edge of the conductor. As a result, opposite charges accumulate on the two side faces of the conductor, as shown in Figure 1.1.

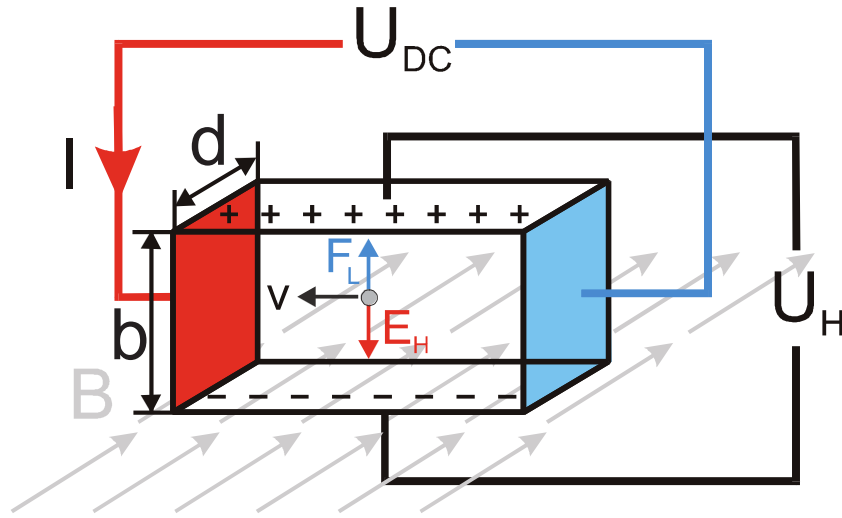


Figure 1.1: Schematic representation of the Hall effect.

The total charge Q is given by

$$Q = n \cdot q, \quad (1.2)$$

where n is the charge carrier density and q the charge of an individual particle.

The force $\vec{F}_k$ resulting from this charge displacement in an electric field $\vec{E}$ is given by

$$\vec{F}_k = Q \cdot \vec{E} = n \cdot q \cdot \vec{E} \quad (1.3)$$

and increases until it compensates the Lorentz force $\vec{F}_L$, so that $\vec{F}_L = -\vec{F}_k$ holds:

$$n \cdot q \cdot \vec{v} \times \vec{B} = -n \cdot q \cdot \vec{E} \quad (1.4)$$

or in the more familiar form

$$n \cdot q \cdot (\vec{E} + \vec{v} \times \vec{B}) = 0. \quad (1.5)$$

We rewrite (1.4) using the current density $\vec{j} = n \cdot q \cdot \vec{v}$ and obtain

$$\vec{j} \times \vec{B} = -n \cdot q \cdot \vec{E}. \quad (1.6)$$

Now we use the relation from electromagnetism

$$\vec{E} = \frac{U}{b} \cdot \vec{e}_b \quad (1.7)$$

and rewrite (1.6) as

$$\vec{j} \times \vec{B} = -n \cdot q \cdot \frac{U_H}{b} \cdot \vec{e}_b. \quad (1.8)$$

This voltage U_H is called the Hall voltage. Solving for U_H gives

$$U_H = -\frac{\vec{j} \times \vec{B}}{n \cdot q} \cdot b \cdot \vec{e}_b \quad (1.9)$$

The vector product $\vec{j} \times \vec{B}$ points in the direction of $\vec{e}_b$ (downward), regardless of the sign of the charge q . Therefore, we can use the magnitudes of $\vec{j} \times \vec{B}$ and $\vec{b}$ to get

$$U_H = -\frac{j \cdot B}{n \cdot q} \cdot b \quad (1.10)$$

Finally, we insert the definition of the current I , given as

$$I = j \cdot A = j \cdot b \cdot d \quad (1.11)$$

with the rectangular cross-sectional area A of the conductor with side lengths b and d . Using $q = e$, the elementary charge, we obtain

$$U_H = -\frac{I \cdot B}{n \cdot e \cdot d} \quad (1.12)$$

1.2.2 The Hall Coefficient

The Hall voltage U_H can also be written as

$$U_H = -\frac{1}{n \cdot e} \cdot \frac{I \cdot B}{d} = A_H \cdot \frac{I \cdot B}{d} \quad (1.13)$$

with the Hall coefficient A_H . By definition, the magnitude of the Hall coefficient is inversely proportional to the charge carrier density. The sign of the Hall coefficient corresponds to the sign of the charge carriers. Therefore, in well-conducting materials, the Hall effect is very small. It becomes significantly more important in semimetals and semiconductors.

subsectionThe Hall Resistance From equation (1.13), the Hall resistance can be determined using Ohm's law

$$U = R \cdot I. \quad (1.14)$$

To do this, equation (1.13) is rearranged as

$$U_H = A_H \cdot \frac{B}{d} \cdot I \quad (1.15)$$

and the proportionality factor between U_H and I is defined as the Hall resistance

$$R_H = A_H \cdot \frac{B}{d} \quad (1.16)$$

1.3 Experiment

1.3.1 Experimental Equipment

Component	Quantity
Hall effect device (silver) (conductor thickness $d = 5 \cdot 10^{-5} \text{m}$)	1
Hall effect device (tungsten) (conductor thickness $d = 5 \cdot 10^{-5} \text{m}$)	1
Microvoltmeter	1
Universal measuring instrument (physics)	1
Combi B-probe S	1
Extension cable, 15-pin	1
Digital multimeter	1
High-current power supply	1
Low-voltage adjustable transformer	1
U-core with yoke	1
Pole shoes, drilled, pair	1
Coil with 250 windings	2
Stand rod 25 cm, 12 mmØ	1
Double clamp	1
V-shaped stand base, small	1
Experiment cable 19 A, 100 cm, red	4
Experiment cable 19 A, 100 cm, blue	4
Experiment cable 32 A, 100 cm, black	2

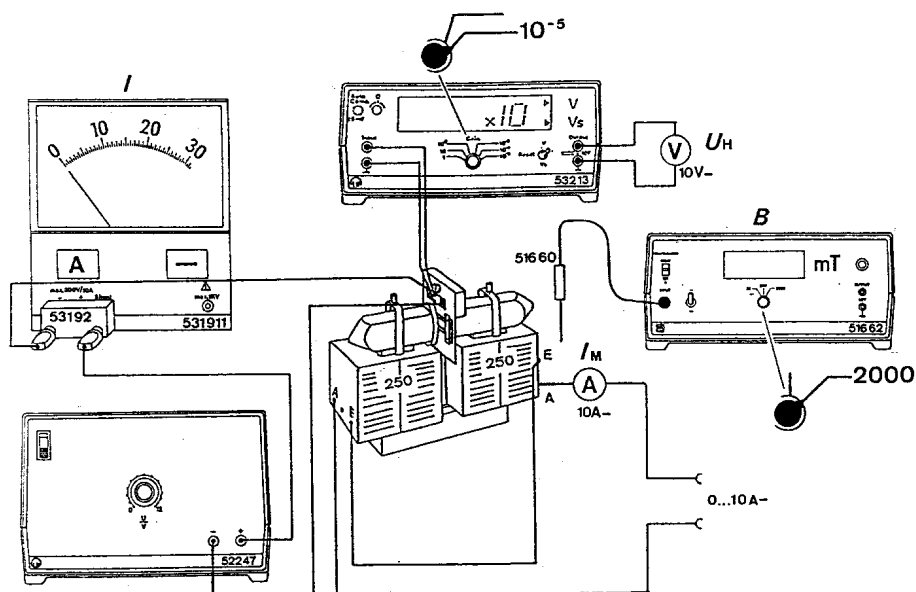


Figure 1.2: Schematic representation of the experimental setup.

1.3.2 Experimental Setup and Calibration

Magnetic Field Calibration

- Set up the calibration experiment according to Figure 1.3 (note: the probe must be rotated 90° along its axis compared to what is shown in the figure). Place the two blue adapters onto the pole shoes and leave them mounted for the entire experiment. For calibration, clamp the red plastic disc between the two pole shoes. The gap created by the plastic disc matches the thickness of the Hall probe, ensuring that the calibration curve is recorded with the same spacing as in the subsequent measurement.
- Connect the two coils in series to the low-voltage adjustable transformer.
- Carefully insert the probe through the opening in the plastic disc into the center between the pole shoes.

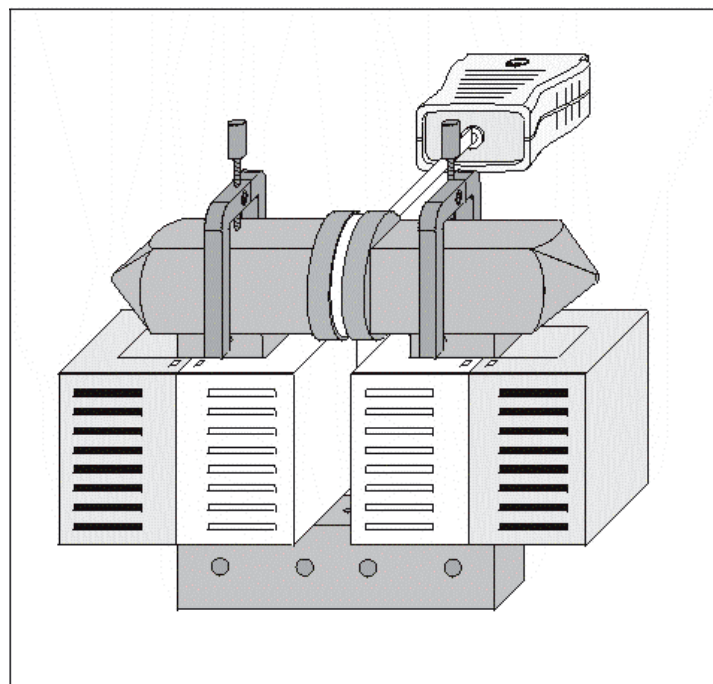


Figure 1.3: Schematic representation of the calibration measurement.

Measurement of the Hall Voltage as a Function of the Magnetic Field

- Remove the probe and the red plastic disc from the electromagnet, and mount the Hall effect device between the two pole shoes. Make sure the pole shoes are pushed as close as possible to the Hall device without damaging it.
- Connect the microvoltmeter to the support plate of the Hall effect device.
- Connect the Hall effect device to the high-current power supply as shown in Figure 1.4. The direction of the magnetic field must match the direction indicated on the device.

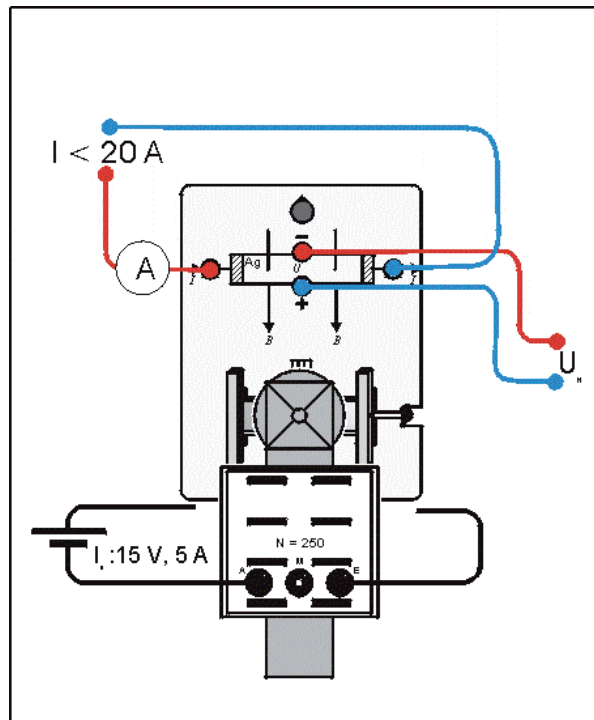


Figure 1.4: Schematic representation of the wiring for Hall effect measurements.

1.3.3 Execution of the Experiment

Calibration of the Magnetic Field

- Demagnetize the iron by allowing the maximum alternating current (AC) to flow through the coils for about 30 seconds, then slowly reduce the current to zero.
- Now switch to direct current (DC). To do this, the cables on the low-voltage adjustable transformer must be reconnected.
- Measure the magnetic flux density B as a function of the current I . From 0 A to 2 A, increase the current in steps of 0.2 A; from 2 A to 9.5 A, increase the current in steps of 0.5 A. Enter your measured values in the table.

Measurement of the Hall Voltage

- Use the high-current power supply to apply a transverse current of $I_t = 10 \text{ A}$.
- Before starting the measurement, the iron must be demagnetized (see above).
- Zero the microvoltmeter («Auto-Komp»-button top left).
- Now measure the Hall voltage U_H as a function of the current I that generates the magnetic field in the coils. The transverse current I_t should remain constant. Enter your measured values into the table.
- Demagnetize and zero again, then repeat this measurement. Try to use the same current values I as in the first run. Fill in your new measurements in the table.
- Increase the transverse current using the high-current power supply to $I_t = 15 \text{ A}$ and repeat steps (2) through (5).

- Now set a transverse current of $I_t = 20, A$ on the high-current power supply and repeat steps (2) through (5). Try to perform this measurement series quickly, as the apparatus heats up significantly at high currents, which can distort the measurement results.
- Now replace the silver Hall effect device with the tungsten Hall effect device and repeat steps (1) through (7). Be careful to connect the tungsten Hall effect device in exactly the same way as the silver one (same cable into the same socket).

1.3.4 Analysis Tasks

Calibration of the Magnetic Field

- Plot your measured values from the magnetic field calibration (with error bars) in a $B(I)$ diagram.
- Determine the magnetic flux density as a function of the current by fitting the function

$$B(I) = a \cdot (1 - e^{b \cdot I}) + c$$

to your measured data points.

- Read off the coefficients a , b , and c . From now on, you can use these to determine the magnetic field for any current I .

Measurement of the Hall Voltage

- Using the calibration above, calculate the corresponding magnetic field B for each current I .
- For both silver and tungsten, two measurement series were taken for each transverse current I_t . Calculate the mean value of the two Hall voltages U_H measured at the same current I , and therefore the same magnetic field B (and same I_t). Provide a reasonable error estimate.
- In a total of six diagrams (three transverse currents I_t each for silver and tungsten), plot the Hall voltage U_H as a function of the magnetic field B . Include error bars in all plots and perform a linear fit.
- Interpret and discuss your plots.
- Determine the Hall constant A_H and the Hall resistance R for both materials at each of the three transverse currents I_t .
- Additionally, calculate the charge carrier concentration n for silver and tungsten for each of the three transverse currents I_t .
- Perform a complete error analysis for the quantities A_H , R , and n .
- Present your results for A_H , R , and n with error estimates clearly in a table. Compare with literature values and interpret your findings.

1.4 Literatur

- D. Meschede, *Gerthsen Physik*, Springer Verlag
- Ashcroft/Mermin, *Solid State Physics*, Cengage Learning Emea
- W. Demtröder, *Experimentalphysik 2*, Springer Verlag
- Wikipedia, www.wikipedia.org/Hall-Konstante, 17.03.2017
- Leybold, <https://www.ld-didactic.de/documents/de-DE/GA/GA/5/586/58681de.pdf>, 18.03.2017

A.1 Measurement tables for the Hall effect

Hall effect

Calibration measurement of the magnetic field

#	I in [A]	B in [mT]
1		
2		
3		
4		
5		
6		
7		
8		

#	I in [A]	B in [mT]
9		
10		
11		
12		
13		
14		
15		
16		

#	I in [A]	B in [mT]
17		
18		
19		
20		
21		
22		
23		
24		

Hall voltage of silver

$I_t = 10A$

#	I in [A]	U_H [$10^{-5}V$]
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		

$I_t = 15A$

#	I in [A]	U_H [$10^{-5}V$]
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		

$I_t = 20A$

#	I in [A]	U_H [$10^{-5}V$]
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		

$I_t = 10A$

#	I in [A]	U_H [$10^{-5}V$]
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		

$I_t = 15A$

#	I in [A]	U_H [$10^{-5}V$]
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		

$I_t = 20A$

#	I in [A]	U_H [$10^{-5}V$]
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		

Hall voltage of tungsten

$$I_t = 10A$$

#	I in [A]	U_H [$10^{-5}V$]
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		

$$I_t = 15A$$

#	I in [A]	U_H [$10^{-5}V$]
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		

$$I_t = 20A$$

#	I in [A]	U_H [$10^{-5}V$]
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		

$$I_t = 10A$$

#	I in [A]	U_H [$10^{-5}V$]
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		

$$I_t = 15A$$

#	I in [A]	U_H [$10^{-5}V$]
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		

$$I_t = 20A$$

#	I in [A]	U_H [$10^{-5}V$]
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		